# Understanding individual human mobility patterns 

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Despite their importance for urban planning ${ }^{1}$, traffic forecasting ${ }^{2}$ and the spread of biological ${ }^{3-5}$ and mobile viruses ${ }^{6}$, our understanding of the basic laws governing human motion remains limited owing to the lack of tools to monitor the time-resolved location of individuals. Here we study the trajectory of 100,000 anonymized mobile phone users whose position is tracked for a six-month period. We find that, in contrast with the random trajectories predicted by the prevailing Lévy flight and random walk models ${ }^{7}$, human trajectories show a high degree of temporal and spatial regularity, each individual being characterized by a timeindependent characteristic travel distance and a significant probability to return to a few highly frequented locations. After correcting for differences in travel distances and the inherent anisotropy of each trajectory, the individual travel patterns collapse into a single spatial probability distribution, indicating that, despite the diversity of their travel history, humans follow simple reproducible patterns. This inherent similarity in travel patterns could impact all phenomena driven by human mobility, from epidemic prevention to emergency response, urban planning and agent-based modelling.

Given the many unknown factors that influence a population's mobility patterns, ranging from means of transportation to job- and family-imposed restrictions and priorities, human trajectories are often approximated with various random walk or diffusion models ${ }^{7,8}$. Indeed, early measurements on albatrosses ${ }^{9}$, followed by more recent data on monkeys and marine predators ${ }^{10,11}$, suggested that animal trajectory is approximated by a Lévy flight ${ }^{12,13}$ —a random walk for which step size $\Delta r$ follows a power-law distribution $P(\Delta r) \sim \Delta r^{-(1+\beta)}$, where the displacement exponent $\beta<2$. Although the Lévy statistics for some animals require further study ${ }^{14}$, this finding has been generalized to humans ${ }^{7}$, documenting that the distribution of distances between consecutive sightings of nearly half-a-million bank notes is fat-tailed. Given that money is carried by individuals, bank note dispersal is a proxy for human movement, suggesting that human trajectories are best modelled as a continuous-time random walk with fat-tailed displacements and waiting-time distributions ${ }^{7}$. A particle following a Lévy flight has a significant probability to travel very long distances in a single step ${ }^{12,13}$, which seems to be consistent with human travel patterns: most of the time we travel only over short distances, between home and work, whereas occasionally we take longer trips.

Each consecutive sighting of a bank note reflects the composite motion of two or more individuals who owned the bill between two reported sightings. Thus, it is not clear whether the observed distribution reflects the motion of individual users or some previously unknown convolution between population-based heterogeneities and individual human trajectories. Contrary to bank notes, mobile phones are carried by the same individual during his/her daily routine, offering the best proxy to capture individual human trajectories ${ }^{15-19}$.

We used two data sets to explore the mobility pattern of individuals. The first $\left(D_{1}\right)$ consisted of the mobility patterns recorded over
a six-month period for 100,000 individuals selected randomly from a sample of more than 6 million anonymized mobile phone users. Each time a user initiated or received a call or a text message, the location of the tower routeing the communication was recorded, allowing us to reconstruct the user's time-resolved trajectory (Fig. 1a, b). The time between consecutive calls followed a 'bursty' pattern ${ }^{20}$ (see Supplementary Fig. 1), indicating that although most consecutive calls are placed soon after a previous call, occasionally there are long periods without any call activity. To make sure that the obtained results were not affected by the irregular call pattern, we also studied a data set $\left(D_{2}\right)$ that captured the location of 206 mobile phone users, recorded every two hours for an entire week. In both data sets, the spatial resolution was determined by the local density of the more than $10^{4}$ mobile towers, registering movement only when the user moved between areas serviced by different towers. The average service area of each tower was approximately $3 \mathrm{~km}^{2}$, and over $30 \%$ of the towers covered an area of $1 \mathrm{~km}^{2}$ or less.

To explore the statistical properties of the population's mobility patterns, we measured the distance between user's positions at consecutive calls, capturing $16,264,308$ displacements for the $D_{1}$ and 10,407 displacements for the $D_{2}$ data set. We found that the distribution of displacements over all users is well approximated by a truncated power-law:

$$
\begin{equation*}
P(\Delta r)=\left(\Delta r+\Delta r_{0}\right)^{-\beta} \exp (-\Delta r / \kappa) \tag{1}
\end{equation*}
$$

with exponent $\beta=1.75 \pm 0.15$ (mean $\pm$ standard deviation), $\Delta r_{0}=1.5 \mathrm{~km}$ and cutoff values $\left.\kappa\right|_{D_{1}}=400 \mathrm{~km}$ and $\left.\kappa\right|_{D_{2}}=80 \mathrm{~km}$ (Fig. 1c, see the Supplementary Information for statistical validation). Note that the observed scaling exponent is not far from $\beta=1.59$ observed in ref. 7 for bank note dispersal, suggesting that the two distributions may capture the same fundamental mechanism driving human mobility patterns.

Equation (1) suggests that human motion follows a truncated Lévy flight ${ }^{7}$. However, the observed shape of $P(\Delta r)$ could be explained by three distinct hypotheses: first, each individual follows a Lévy trajectory with jump size distribution given by equation (1) (hypothesis A); second, the observed distribution captures a population-based heterogeneity, corresponding to the inherent differences between individuals (hypothesis B); and third, a population-based heterogeneity coexists with individual Lévy trajectories (hypothesis C); hence, equation (1) represents a convolution of hypotheses A and B.

To distinguish between hypotheses A, B and C, we calculated the radius of gyration for each user (see Supplementary Information), interpreted as the characteristic distance travelled by user $a$ when observed up to time $t$ (Fig. 1b). Next, we determined the radius of gyration distribution $P\left(r_{\mathrm{g}}\right)$ by calculating $r_{\mathrm{g}}$ for all users in samples $D_{1}$ and $D_{2}$, finding that they also can be approximated with a truncated power-law:

$$
\begin{equation*}
P\left(r_{\mathrm{g}}\right)=\left(r_{\mathrm{g}}+r_{\mathrm{g}}^{0}\right)^{-\beta_{r}} \exp \left(-r_{\mathrm{g}} / \kappa\right) \tag{2}
\end{equation*}
$$

[^0]with $r_{\mathrm{g}}^{0}=5.8 \mathrm{~km}, \beta_{r}=1.65 \pm 0.15$ and $\kappa=350 \mathrm{~km}$ (Fig. 1d, see Supplementary Information for statistical validation). Lévy flights are characterized by a high degree of intrinsic heterogeneity, raising the possibility that equation (2) could emerge from an ensemble of identical agents, each following a Lévy trajectory. Therefore, we determined $P\left(r_{\mathrm{g}}\right)$ for an ensemble of agents following a random walk (RW), Lévy flight (LF) or truncated Lévy flight (TLF) (Fig. 1d) ${ }^{8,12,13}$. We found that an ensemble of Lévy agents display a significant degree of heterogeneity in $r_{g}$; however, this was not sufficient to explain the truncated power-law distribution $P\left(r_{\mathrm{g}}\right)$ exhibited by the mobile phone users. Taken together, Fig. 1c and d suggest that the difference in the range of typical mobility patterns of individuals $\left(r_{\mathrm{g}}\right)$ has a strong impact on the truncated Lévy behaviour seen in equation (1), ruling out hypothesis A.

If individual trajectories are described by an LF or TLF, then the radius of gyration should increase with time as $r_{\mathrm{g}}(t) \sim t^{3 /(2+\beta)}$ (ref. 21), whereas, for an RW, $r_{\mathrm{g}}(t) \sim t^{1 / 2}$; that is, the longer we observe a user, the higher the chance that she/he will travel to areas not visited before. To check the validity of these predictions, we measured the time dependence of the radius of gyration for users whose gyration radius would be considered small $\left(r_{\mathrm{g}}(T) \leq 3 \mathrm{~km}\right)$, medium $\left(20<r_{\mathrm{g}}(T) \leq 30 \mathrm{~km}\right)$ or large $\left(r_{\mathrm{g}}(T)>100 \mathrm{~km}\right)$ at the end of our observation period ( $T=6$ months). The results indicate that
the time dependence of the average radius of gyration of mobile phone users is better approximated by a logarithmic increase, not only a manifestly slower dependence than the one predicted by a power law but also one that may appear similar to a saturation process (Fig. 2a and Supplementary Fig. 4).

In Fig. 2b, we chose users with similar asymptotic $r_{\mathrm{g}}(T)$ after $T=6$ months, and measured the jump size distribution $P\left(\Delta r \mid r_{\mathrm{g}}\right)$ for each group. As the inset of Fig. 2b shows, users with small $r_{\mathrm{g}}$ travel mostly over small distances, whereas those with large $r_{\mathrm{g}}$ tend to display a combination of many small and a few larger jump sizes. Once we rescaled the distributions with $r_{\mathrm{g}}$ (Fig. 2b), we found that the data collapsed into a single curve, suggesting that a single jump size distribution characterizes all users, independent of their $r_{\mathrm{g}}$. This indicates that $P\left(\Delta r \mid r_{\mathrm{g}}\right) \sim r_{\mathrm{g}}^{-\alpha} F\left(\Delta r / r_{\mathrm{g}}\right)$, where $\alpha \approx 1.2 \pm 0.1$ and $F(x)$ is an $r_{\mathrm{g}}$-independent function with asymptotic behaviour, that is, $F(x) \sim x^{-\alpha}$ for $x<1$ and $F(x)$ rapidly decreases for $x \gg 1$. Therefore, the travel patterns of individual users may be approximated by a Lévy flight up to a distance characterized by $r_{\mathrm{g}}$. Most important, however, is the fact that the individual trajectories are bounded beyond $r_{g}$; thus, large displacements, which are the source of the distinct and anomalous nature of Lévy flights, are statistically absent. To understand the relationship between the different exponents, we note that the measured probability distributions are related


Figure $1 \mid$ Basic human mobility patterns. a, Week-long trajectory of 40 mobile phone users indicates that most individuals travel only over short distances, but a few regularly move over hundreds of kilometres. $\mathbf{b}$, The detailed trajectory of a single user. The different phone towers are shown as green dots, and the Voronoi lattice in grey marks the approximate reception area of each tower. The data set studied by us records only the identity of the closest tower to a mobile user; thus, we can not identify the position of a user within a Voronoi cell. The trajectory of the user shown in $\mathbf{b}$ is constructed from 186 two-hourly reports, during which the user visited a total of 12 different locations (tower vicinities). Among these, the user is found on 96 and 67 occasions in the two most preferred locations; the frequency of visits
for each location is shown as a vertical bar. The circle represents the radius of gyration centred in the trajectory's centre of mass. c, Probability density function $P(\Delta r)$ of travel distances obtained for the two studied data sets $D_{1}$ and $D_{2}$. The solid line indicates a truncated power law for which the parameters are provided in the text (see equation (1)). $\mathbf{d}$, The distribution $P\left(r_{\mathrm{g}}\right)$ of the radius of gyration measured for the users, where $r_{\mathrm{g}}(T)$ was measured after $T=6$ months of observation. The solid line represents a similar truncated power-law fit (see equation (2)). The dotted, dashed and dot-dashed curves show $P\left(r_{\mathrm{g}}\right)$ obtained from the standard null models (RW, LF and TLF, respectively), where for the TLF we used the same step size distribution as the one measured for the mobile phone users.
by $P(\Delta r)=\int_{0}^{\infty} P\left(\Delta r \mid r_{\mathrm{g}}\right) P\left(r_{\mathrm{g}}\right) d r_{\mathrm{g}}$, which suggests (see Supplementary Information) that up to the leading order we have $\beta=\beta_{r}+\alpha-1$, consistent, within error bars, with the measured exponents. This indicates that the observed jump size distribution $P(\Delta r)$ is in fact the convolution between the statistics of individual trajectories $P\left(\Delta r_{\mathrm{g}} \mid r_{\mathrm{g}}\right)$ and the population heterogeneity $P\left(r_{\mathrm{g}}\right)$, consistent with hypothesis C .

To uncover the mechanism stabilizing $r_{g}$, we measured the return probability for each individual $F_{\mathrm{pt}}(t)$ (first passage time probability $)^{21,22}$, defined as the probability that a user returns to the position where he/she was first observed after $t$ hours (Fig. 2c). For a two-dimensional random walk, $F_{\mathrm{pt}}(t)$ should follow $\sim 1 /\left(t \ln ^{2}(t)\right)$ (ref. 21). In contrast, we found that the return probability is characterized by several peaks at $24 \mathrm{~h}, 48 \mathrm{~h}$ and 72 h , capturing a strong tendency of humans to return to locations they visited before, describing the recurrence and temporal periodicity inherent to human mobility ${ }^{23,24}$.


Figure 2 | The bounded nature of human trajectories. a, Radius of gyration $\left\langle r_{\mathrm{g}}(t)\right\rangle$ versus time for mobile phone users separated into three groups according to their final $r_{\mathrm{g}}(T)$, where $T=6$ months. The black curves correspond to the analytical predictions for the random walk models, increasing with time as $\left.\left\langle r_{\mathrm{g}}(t)\right\rangle\right|_{\text {LF,TLF }} \sim t^{3 / 2+\beta}$ (solid curve) and $\left.\left\langle r_{\mathrm{g}}(t)\right\rangle\right|_{\mathrm{RW}} \sim t^{0.5}$ (dotted curve). The dashed curves corresponding to a logarithmic fit of the form $A+B \ln (t)$, where $A$ and $B$ are time-independent coefficients that depend on $r_{g} \cdot b$, Probability density function of individual travel distances $P\left(\Delta r \mid r_{\mathrm{g}}\right)$ for users with $r_{\mathrm{g}}=4,10,40,100$ and 200 km . As the inset shows, each group displays a quite different $P\left(\Delta r \mid r_{\mathrm{g}}\right)$ distribution. After rescaling the distance and the distribution with $r_{\mathrm{g}}$ (main panel), the different curves collapse. The solid line (power law) is shown as a guide to the eye.
c, Return probability distribution, $F_{\mathrm{pt}}(t)$. The prominent peaks capture the tendency of humans to return regularly to the locations they visited before, in contrast with the smooth asymptotic behaviour $\sim 1 /\left(t \ln (t)^{2}\right)$ (solid line) predicted for random walks. d, A Zipf plot showing the frequency of visiting different locations (loc.). The symbols correspond to users that have been observed to visit $n_{L}=5,10,30$ and 50 different locations. Denoting with $(L)$, the rank of the location listed in the order of the visit frequency, the data are well approximated by $R(L) \sim L^{-1}$. The inset is the same plot in linear scale, illustrating that $40 \%$ of the time individuals are found at their first two preferred locations; bars indicate the standard error.

To explore if individuals return to the same location over and over, we ranked each location on the basis of the number of times an individual was recorded in its vicinity, such that a location with $L=3$ represents the third-most-visited location for the selected individual. We find that the probability of finding a user at a location with a given rank $L$ is well approximated by $P(L) \sim 1 / L$, independent of the number of locations visited by the user (Fig. 2d). Therefore, people devote most of their time to a few locations, although spending their remaining time in 5 to 50 places, visited with diminished regularity. Therefore, the observed logarithmic saturation of $r_{\mathrm{g}}(t)$ is rooted in the high degree of regularity in the daily travel patterns of individuals, captured by the high return probabilities (Fig. 2b) to a few highly frequented locations (Fig. 2d).

An important quantity for modelling human mobility patterns is the probability density function $\Phi_{a}(x, y)$ to find an individual $a$ in a given position $(x, y)$. As it is evident from Fig. 1b, individuals live and travel in different regions, yet each user can be assigned to a well defined area, defined by home and workplace, where she or he can be found most of the time. We can compare the trajectories of different users by diagonalizing each trajectory's inertia tensor, providing the probability of finding a user in a given position (see Fig. 3a) in the user's intrinsic reference frame (see Supplementary Information for the details). A striking feature of $\Phi(x, y)$ is its prominent spatial anisotropy in this intrinsic reference frame (note the different scales in Fig. 3a); we find that the larger an individual's $r_{\mathrm{g}}$, the more pronounced is this anisotropy. To quantify this effect, we defined the anisotropy ratio $S \equiv \sigma_{y} / \sigma_{x}$, where $\sigma_{x}$ and $\sigma_{y}$ represent the standard deviation of the trajectory measured in the user's intrinsic reference frame (see Supplementary Information). We found that $S$ decreases monotonically with $r_{\mathrm{g}}$ (Fig. 3c), being well approximated with $S \sim r_{\mathrm{g}}^{-\eta}$ for $\eta \approx 0.12$. Given the small value of the scaling exponent, other functional forms may offer an equally good fit; thus, mechanistic models are required to identify if this represents a true scaling law or only a reasonable approximation to the data.

To compare the trajectories of different users, we removed the individual anisotropies, rescaling each user trajectory with its respective $\sigma_{x}$ and $\sigma_{y}$. The rescaled $\tilde{\Phi}\left(x / \sigma_{x}, y / \sigma_{y}\right)$ distribution (Fig. 3b) is similar for groups of users with considerably different $r_{\mathrm{g}}$, that is, after the anisotropy and the $r_{\mathrm{g}}$ dependence are removed all individuals seem to follow the same universal $\tilde{\Phi}(\tilde{x}, \tilde{y})$ probability distribution. This is particularly evident in Fig. 3d, where we show the cross section of $\tilde{\Phi}\left(x / \sigma_{x}, 0\right)$ for the three groups of users, finding that apart from the noise in the data the curves are indistinguishable.

Taken together, our results suggest that the Lévy statistics observed in bank note measurements capture a convolution of the population heterogeneity shown in equation (2) and the motion of individual users. Individuals display significant regularity, because they return to a few highly frequented locations, such as home or work. This regularity does not apply to the bank notes: a bill always follows the trajectory of its current owner; that is, dollar bills diffuse, but humans do not.

The fact that individual trajectories are characterized by the same $r_{\mathrm{g}}$-independent two-dimensional probability distribution $\tilde{\Phi}\left(x / \sigma_{x}, y / \sigma_{y}\right)$ suggests that key statistical characteristics of individual trajectories are largely indistinguishable after rescaling. Therefore, our results establish the basic ingredients of realistic agent-based models, requiring us to place users in number proportional with the population density of a given region and assign each user an $r_{\mathrm{g}}$ taken from the observed $P\left(r_{\mathrm{g}}\right)$ distribution. Using the predicted anisotropic rescaling, combined with the density function $\tilde{\Phi}(x, y)$, the shape of which is provided as Supplementary Table 1, we can obtain the likelihood of finding a user in any location. Given the known correlations between spatial proximity and social links, our results could help quantify the role of space in network development and evolution ${ }^{25-29}$ and improve our understanding of diffusion processes ${ }^{8,30}$.


Figure 3 | The shape of human trajectories.
a, The probability density function $\Phi(x, y)$ of finding a mobile phone user in a location $(x, y)$ in the user's intrinsic reference frame (see Supplementary Information for details). The three plots, from left to right, were generated for 10,000 users with: $r_{\mathrm{g}} \leq 3,20<r_{\mathrm{g}} \leq 30$ and $r_{\mathrm{g}}>100 \mathrm{~km}$. The trajectories become more anisotropic as $r_{\mathrm{g}}$ increases. $\mathbf{b}$, After scaling each position with $\sigma_{x}$ and $\sigma_{y}$, the resulting $\tilde{\Phi}\left(x / \sigma_{x}, y / \sigma_{y}\right)$ has approximately the same shape for each group. c, The change in the shape of $\Phi(x, y)$ can be quantified calculating the isotropy ratio $S \equiv \sigma_{y} / \sigma_{x}$ as a function of $r_{g}$, which decreases as $S \sim r_{\mathrm{g}}^{-0.12}$ (solid line). Error bars represent the standard error. d, $\tilde{\Phi}\left(x / \sigma_{x}, 0\right)$ representing the $x$-axis cross-section of the rescaled distribution $\tilde{\Phi}\left(x / \sigma_{x}, y / \sigma_{y}\right)$ shown in $\mathbf{b}$.

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## Supplementary Material

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## I. DATA

A. $D_{1}$ Dataset: This dataset was collected by a European mobile phone carrier for billing and operational purposes. It contains the date, time and coordinates of the phone tower routing the communication for each phone call and text message sent or received by 6 million costumers. The dataset summarizes 6 months of activity. To guarantee anonymity, each user is identified with a security key (hash code). Furthermore, we only know the coordinates of the tower routing the communication, hence a user's location is not known within a tower's service area. Each tower serves an area of approximately $3 \mathrm{~km}^{2}$. Due to tower coverage limitations driven by geographical constraints and national frontiers no jumps exceeding $\sim 1,000 \mathrm{~km}$ can be observed in the dataset.

The research was performed on a random set of 100,000 selected from those making or receiving at least one phone call or SMS during the first and last month of the study, translating to $16,364,308$ recorded positions. We removed all jumps that took users outside the continental territory. We did not impose any additional criterion regarding the calling activity to avoid possible selection biases in the mobility pattern.
B. $D_{2}$ Dataset: Some services provided by the mobile phone carrier, like pollen and traffic forecasts, rely on the approximate knowledge of customer's location at all times of the day. For customers that signed up for location dependent services, the date, time and the closest tower coordinates are recorded on a regular basis, independent of their phone usage. We were provided such records for 1,000 users, among which we selected the group of users whose coordinates were recorded at every two hours during an entire week, resulting in 206 users for which we have 10,613 recorded positions. Given that these users were selected based on their actions (signed up to the service), in principle the sample cannot be considered unbiased, but we have not detected any particular bias for this data set.

For each user in $D_{1}$ and $D_{2}$ we sorted the time resolved sequence of positions and constructed individual trajectories.


FIG. S1: Interevent time distribution $P(\Delta T)$ of calling activity. $\Delta T$ is the time elapsed between consecutive communication records (phone calls and SMS, sent or received) for the same user. Different symbols indicate the measurements done over groups of users with different activity levels (\# calls). The inset shows the unscaled version of this plot. The solid line corresponds to Eq. (S1).

## II. CHARACTERIZING INDIVIDUAL CALLING ACTIVITY

Communication patterns are known to be highly heterogeneous: some users rarely use the mobile phone while others make hundreds or even thousands of calls each month [1]. To characterize the dynamics of individual communication activity, we grouped users based on their total number of calls. For each user we measured the probability that the time interval between two consecutive calls is $\Delta T$ [2-4]. The inset of Fig. S1 shows that users with less activity tend to have longer waiting times between consecutive calls. By rescaling the axis with the average interevent time $\Delta T_{a}$ as $\Delta T_{a} P(\Delta T)$ and $\Delta T / \Delta T_{a}$ the obtained distributions collapse into a single curve (Fig. S1). Hence the measured interevent time distribution can be approximated by the expression $P(\Delta T)$
$=1 / \Delta T_{a} \mathcal{F}\left(\Delta T / \Delta T_{a}\right)$, where $\mathcal{F}(x)$ is independent of the average activity level of the population. This is a universal characteristic of the system and it agrees with earlier results on the temporal patterns of e-mail communication [5]. In addition, we find that the data in Fig. S1 is well approximated by

$$
\begin{equation*}
P(\Delta T)=(\Delta T)^{\alpha} \exp \left(\Delta T / \tau_{c}\right) \tag{S1}
\end{equation*}
$$

where the power-law exponent $\alpha=0.9 \pm 0.1$ is followed by an exponential cutoff of $\tau_{c} \approx 48$ days. Equation (S1) is shown by a solid line in the inset of Fig. S1 and its scaled version is presented in the main panel of Fig. S1. Here we used $\Delta T_{a}=8.2$ hours, which is the average interevent time measured for the whole population. The heterogeneity in the communication pattern translates into heterogeneous sampling for the $D_{1}$ dataset. The $D_{2}$ dataset, with records at every two hours, obviously does not display this heterogeneity. Below we show that this temporal heterogeneity does not affect our results on the observed travel patterns.

## III. OBSERVATIONS AT A FIXED INTEREVENT TIME

Given the widely varying distribution of the interevent times between two calls (an therefore the localization data), we need to investigate if the observed displacement statistics are affected by this sampling heterogeneity. Using the $D_{1}$ dataset, we calculated the displacement distribution $P(\Delta r)$ for consecutive calls separated by a time $\Delta T_{o} \pm 0.05 \Delta T_{o}$, where $\Delta T_{o}$ ranged from 20 min to one day. Figure S 2 shows that for $\Delta T_{o}<4 \mathrm{~h}$, the observed displacements are bounded by the maximum distance that users can travel in the $\Delta T_{o}$ time interval. For $\Delta T_{o} \geq 8$ hours we already observe $\Delta r_{\max } \sim 1,000 \mathrm{~km}$, which corresponds to the largest displacement we could possible observe given the area under study (such large jumps likely are the result of airline travel). We observe that the resulting $P(\Delta r)$ distributions for different $\Delta T_{o}$ is again well approximated by a truncated power-law with an exponent $\beta=1.75$. This agrees with the exponent found when we studied all consecutive calls (see Fig. 1C), suggesting that the use of consecutive calls is an accurate proxy to measure human displacement at large enough scales ( $>1 \mathrm{~km}$ ).


FIG. S2: Displacement distribution $P(\Delta r)$ for fixed inter event times $\Delta T_{o}$ based on the $D_{1}$ dataset. The cutoff of the distribution is set by the maximum distance users can travel for shorter inter event times, whereas for longer times the cutoff is given by the finite size of the studied area, as discussed in the manuscript. The black line is from (1) reported in the manuscript, with the value $\kappa=400 \mathrm{~km}$ corresponding to $D_{1}$ (solid line).

## IV. INTRINSIC REFERENCE FRAME FOR INDIVIDUAL TRAJECTORIES

A. Radius of gyration: The linear size occupied by each user's trajectory up to time $t$ is characterized by its the radius of gyration defined as

$$
\begin{equation*}
r_{g}^{a}(t)=\sqrt{\frac{1}{n_{c}^{a}(t)} \sum_{i=1}^{n_{c}^{a}}\left(\overrightarrow{r_{i}^{a}}-\vec{r}_{c m}^{a}\right)^{2}} \tag{S2}
\end{equation*}
$$

where $\vec{r}_{i}^{a}$ represents the $i=1, \ldots, n_{c}^{a}(t)$ positions recorded for user $a$ and $\vec{r}_{c m}^{a}=1 / n_{c}^{a}(t) \sum_{i=1}^{n_{c}^{a}} \overrightarrow{r_{i}^{a}}$ is the center of mass of the trajectory.
B. Moment of Inertia: To compare different users' trajectories we need to study them in a common reference frame. Inspired by the mechanics of rigid bodies [6], we assign each user to an intrinsic reference frame calculated a posteriori from a user's trajectory. We can think of the number times a user visited a given location as the mass associated with that particular position. We denote a user's trajectory with a set of locations $\left\{\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots\left(x_{n_{c}}, y_{n_{c}}\right)\right\}$, where $n_{c}$ is the number of positions available for the user. An object's moment of inertia is given by the average spread of an object's mass from a given axis. A two dimensional object can be characterized by a $2 \times 2$ matrix known as the tensor of inertia

$$
\mathbf{I}=\left(\begin{array}{cc}
I_{x x} & I_{x y}  \tag{S3}\\
I_{y x} & I_{y y}
\end{array}\right)
$$

We can calculate the inertia tensor for user's trajectories by using the standard physical formulas

$$
\begin{align*}
I_{x x} & \equiv \sum_{i=1}^{n_{c}} y_{i}^{2}  \tag{S4}\\
I_{y y} & \equiv \sum_{i=1}^{n_{c}} x_{i}^{2}  \tag{S5}\\
I_{x y}=I_{y x} & \equiv-\sum_{i=1}^{n_{c}} x_{i} y_{i} . \tag{S6}
\end{align*}
$$

Since the tensor $\mathbf{I}$ is symmetric, it is possible to find a set of coordinates in which $\mathbf{I}$ will be diagonal. These coordinates are known as the tensor's principal axes $\left(\hat{e}_{1}, \hat{e}_{2}\right)$. In this set of coordinates I takes the form

$$
\mathbf{I}_{\mathbf{D}}=\left(\begin{array}{cc}
I_{1} & 0  \tag{S7}\\
0 & I_{2}
\end{array}\right)
$$

where $I_{1}$ and $I_{2}$ are the principal moments of inertia. They also correspond to the eigenvalues of $I$ and can be calculated from the original set of points as

$$
\begin{align*}
& I_{1}=\frac{1}{2}\left(I_{x x}+I_{y y}\right)-\frac{1}{2} \mu  \tag{S8}\\
& I_{2}=\frac{1}{2}\left(I_{x x}+I_{y y}\right)+\frac{1}{2} \mu, \tag{S9}
\end{align*}
$$

with

$$
\begin{equation*}
\mu \equiv \sqrt{4 I_{x y} I_{y x}+I_{x x}{ }^{2}-2 I_{x x} I_{y y}+I_{y y}{ }^{2}} \tag{S10}
\end{equation*}
$$

The corresponding eigenvectors determine the principal axes ( $\hat{e}_{1}$ and $\hat{e}_{2}$ ), representing the symmetry axes of a given trajectory.
C. Rotation of user trajectories: We transform each user's principal axes ( $\hat{e}_{1}, \hat{e}_{2}$ ) to a common intrinsic reference frame $\left(\hat{e}_{x}, \hat{e}_{y}\right)$ calculating the angle between the axes $\hat{e}_{x}$ and $\hat{e}_{1}$, as

$$
\begin{equation*}
\cos (\theta)=\frac{\vec{v}_{1} \cdot \hat{e}_{x}}{\left|\vec{v}_{1}\right|} \tag{S11}
\end{equation*}
$$

where $v_{1}$, is the eigenvector associated with the eigenvalue $I_{1}$

$$
\vec{v}_{1}=\left[\begin{array}{c}
-\frac{I_{x y}}{1 / 2 I_{x x}-1 / 2 I_{y y}+1 / 2 \mu}  \tag{S12}\\
1
\end{array}\right],
$$

resulting in

$$
\begin{equation*}
\cos (\theta)=-I_{x y}\left(1 / 2 I_{x x}-1 / 2 I_{y y}+1 / 2 \mu\right)^{-1} \frac{1}{\sqrt{1+\frac{I_{x y^{2}}}{\left(1 / 2 I_{x x}-1 / 2 I_{y y}+1 / 2 \mu\right)^{2}}}} . \tag{S13}
\end{equation*}
$$

After rotation by $\theta$, we impose a conditional rotation of $180^{\circ}$ such that the most frequent position lays always in $x>0$.
D. Example: Figure S3 shows the recorded trajectories of 3 users ( $u_{1}, u_{2}$ and $u_{3}$ ), each characterized by a different radius of gyration: $\left.r_{g}\right|_{u 1}=2.28 \mathrm{~km},\left.r_{g}\right|_{u 2}=29.02 \mathrm{~km}$, and $\left.r_{g}\right|_{u 3}=313.72$ km . Using (S4), (S5) and (S6), we calculated the different components of the tensor of inertia. Equations (S12) and (S13) allow us to determine the intrinsic axes for each user ( $\hat{e}_{1}, \hat{e}_{2}$ ), which are displayed in Fig. S3a. Their respective angles are: $\left.\theta\right|_{u 1}=127.67^{\circ},\left.\theta\right|_{u 2}=40.20^{\circ}$ and $\left.\theta\right|_{u 3}=60.08^{\circ}$. Each set of points is rotated by $-\theta$, such that $\left(\hat{e}_{x}, \hat{e}_{y}\right)$ is the new intrinsic reference frame of each user's trajectory, as shown in Fig. S3b. The most frequent and the second most frequent positions of each user are marked as a blue and orange circle respectively. After rotating the trajectory of user 2, its most frequent position lays in $x<0$, hence we apply an additional rotation of $180^{\circ}$ such that the most frequent position lays in $x>0$. The purpose of this is to conserve the asymmetry of the user's visitation pattern. In the absence of the rotation the trajectories in Fig. S3a and B (also Fig. 3 in the manuscript) will appear to be symmetric. Given, however, that there is a significant difference in the most and the second most visited locations (see Fig. 2D in the paper), we need to perform the symmetry breaking rotation to emphasize its presence. For example, we


FIG. S3: Example of how to transform the user trajectories in a common reference frame. a, Initial trajectories of three users and their principal axes ( $\hat{e}_{1}, \hat{e}_{2}$ ). b, Each trajectory is rotated an angle $-\theta$ to align $\hat{e}_{1}$ with $\hat{e}_{x}$. An additional rotation with $180^{\circ}$ is required when the most frequent position (marked with a blue circle) lays in $x<0$ after the rotation. This is the case of user 2 (green line). $\mathbf{c}$, Positions $(x, y)$ are scaled as $\left(x / \sigma_{x}, y / \sigma_{y}\right)$ after which the different trajectories have a quite similar shape.
found that for finite Lévy flights the rotation induced a slight but detectable anisotropy, capturing the fact that each finite trajectory has some inherent anisotropy.

We scale the trajectories on the intrinsic axes with the standard deviation of the locations for each user $a$

$$
\begin{align*}
& \sigma_{x}^{a}=\sqrt{\frac{1}{n_{c}^{a}} \sum_{i=1}^{n_{c}^{a}}\left(x_{i}^{a}-x_{c m}^{a}\right)^{2}},  \tag{S14}\\
& \sigma_{y}^{a}=\sqrt{\frac{1}{n_{c}^{a}} \sum_{i=1}^{n_{c}^{a}}\left(y_{i}^{a}-y_{c m}^{a}\right)^{2} .} \tag{S15}
\end{align*}
$$

Note that the coordinate origin for each user is placed at the center of mass of the trajectory, $r_{c m}^{\vec{a}}=(0,0)$. In this example, $\left.\sigma_{x}\right|_{u 1}=2.24 \mathrm{~km},\left.\sigma_{x}\right|_{u 2}=28.76 \mathrm{~km}$, and $\left.\sigma_{x}\right|_{u 3}=313.60 \mathrm{~km}$ whereas $\left.\sigma_{y}\right|_{u 1}=0.43 \mathrm{~km},\left.\sigma_{y}\right|_{u 2}=3.88 \mathrm{~km}$, and $\left.\sigma_{y}\right|_{u 3}=8.49 \mathrm{~km}$. After scaling, the shapes of the three trajectories look similar (S3c), despite that we are showing users with significantly different mobility patterns and ranges. This is the underlying procedure that allows us to obtain
the universal density function $\tilde{\Phi}\left(x / \sigma_{x}, y / \sigma_{y}\right)$.
E. Spatial density function: For agent based modeling it is crucial to know the probability that an individual can be found at a position $(x, y)$ during the day. As our results show, knowledge of the spatial density function $\tilde{\Phi}\left(x / \sigma_{x}, y / \sigma_{y}\right)$ represent the first step towards such a modeling effort. Indeed, using the density function $\tilde{\Phi}\left(x / \sigma_{x}, y / \sigma_{y}\right)$ for an ensemble of agents with $r_{g}$ 's following Eq.(3), each agent's position can be rescaled using Eq.(4) and the fact that $\sigma_{x}=0.94 r_{g}^{0.97}$. The distribution of individuals in space can be arbitrary or more realistic if taken from census information. The three matrixes shown in Fig. 3B can be downloaded from: http://www.nd.edu/ mgonza16/DensityFunction/

## V. SCALING RELATION BETWEEN EXPONENTS

Next we show that there is a consistent relationship among the different exponents describing the travel patterns of the population. The exponent $\beta$ characterizing the distances traveled by the entire population is related to $\alpha$, which characterizes distances traveled by individuals and $\beta_{r}$, that captures the distribution of the radius of gyration. We note that (1) should be the result of a convolution between (3) and $P\left(\Delta r \mid r_{g}\right)$, hence

$$
\begin{equation*}
P(\Delta r)=\int_{0}^{\infty} P\left(\Delta r \mid r_{g}\right) P\left(r_{g}\right) d r_{g} \tag{S16}
\end{equation*}
$$

using the expressions introduced in the manuscript this equation can be expanded as

$$
\begin{equation*}
P(\Delta r)=\int_{0}^{\infty} r_{g}^{-\alpha} F\left(\frac{\Delta r}{r_{g}}\right)\left(r_{g}+r_{g}^{0}\right) e^{-r_{g} / \kappa} d r_{g} . \tag{S17}
\end{equation*}
$$

Focusing on the asymptotic scaling behavior we drop the short length cutoff $r_{g}^{0}$ and extract the leading term by performing the substitution $r_{g}=\Delta r x$. Finally the scaling is given by

$$
\begin{equation*}
P(\Delta r) \approx \Delta r^{-\alpha-\beta_{r}+1} \int_{0}^{\infty} x^{-\alpha} F\left(\frac{1}{x}\right) e^{x \Delta r / \kappa} d x \tag{S18}
\end{equation*}
$$

indicating that $\beta=\alpha+\beta_{r}-1$. Note, however, that the integral in (S18) also depends on $\Delta r$, therefore the scaling relationship is valid only to the leading order and further corrections may result from the integral. This correction cannot be evaluated in the absence of an analytical approximation for $F(x)$. For our data we find $\beta=1.75 \pm 0.15, \beta_{r}=1.65 \pm 0.15$ and $\alpha=1.2 \pm 0.1$, indicating that the scaling relation, within error bars, is satisfied, and that there is a systematic difference between $\beta$ and $\beta_{r}$ of magnitude $\alpha-1$.

## VI. TIME DEPENDENCE OF THE RADIUS OF GYRATION



FIG. S4: Time evolution of the radius of gyration $\left\langle r_{g}(t)\right\rangle$ vs. time, for various groups of users with different asymptotic $r_{g}(T)$ after $T=6$ months. a, In a $\log$-log scale the black lines correspond to the powers of time for the random walk and Lévy flight models, which are in contrast with the time dependence of $\left\langle r_{g}(t)\right\rangle$ measured for the mobile phone users. b, In a log-linear scale, note that the $r_{g}=4$ and $r_{g}=10$ visibly deviate from the large $r_{g}$ curves. This is not surprising, as for these two curves the recorded distances are comparable to the average tower distances (both curves appear to saturate under 10 km , while the average area of reception for a tower is about $3 \mathrm{~km}^{2}$ ). Thus, small travel distances are overestimated due to the measurement resolution. Curves with $r_{g}>10 \mathrm{~km}$ are less affected by the tower resolution and all these appear to collapse in the same behavior once rescaled with $r_{g}(T)$, and they are all approximated with a logarithmic time dependence. The straight line is not a fit, but it is shown only as a guide to the eye.

Figure 2A in the manuscript shows three groups of users chosen according to the asymptotic $r_{g}(T)$ after $T=6$ months. In Fig. S4 we show that the same time dependence is observed for a more strictly selected grouping of the users, choosing five different groups of users with very similar asymptotic radius of gyration: $r_{g}(T) \pm 0.05 r_{g}(T)$. Given the high daily and weekly-based
fluctuations in the phone usage patterns, we averaged $\left\langle r_{g}(t)\right\rangle$ over 168 different initial conditions, i.e. we started the measurements at every 6 hours during one week. This averaging not only removed the dependence on the initial conditions, but also significantly reduced the noise in the curves.

The log-log scale in Fig. S4a allows to see in detail the early behavior of the curves, indicating that a power law does not offer a good fit to the data. As we show in the log-linear plot in Fig. S4b, we find that the radius of gyration increases logarithmically in time, which is in strong contrast with the power law dependence expected for Lévy flights $\left(\left.r_{g}(t)\right|_{L F / T L F} \sim t^{3 /(2+\beta)}\right)$ and random walks $\left(\left.r_{g}(t)\right|_{R W} \sim t^{1 / 2}\right)$. This indicates that the average radius of gyration of mobile phone users has a manifestly slower dependence than the predicted power laws, a behavior that may appear similar to a saturation process. Note that the $r_{g} \sim 4,10$ curves appear to deviate from the logarithmic behavior. We belive that this is due to the spatial resolution offered by the tower density: we cannot reliably and systematically resolve jumps in the vicinity of a few kms, given that we record a motion only when a person moves between towers, that are often a few kilometers apart. The $r_{g}>10 \mathrm{~km}$ curves, given the distances involved, are not affected by the granularity of the data collection process, and they all follow the logarithmic behavior.

## VII. STATISTICAL TESTS OF FITTING DISTRIBUTIONS

Given the fat tailed distributions observed for human travel patterns, it is important to see if the data is statistically consistent with the best fits. The purpose of this section is to support our findings with rigorous statistical tests. In the past year there has been significant attention devoted to the question of how to statistically measure the goodness of fit for a power law [7-10]. This was prompted partly by the need to quantify the validity of the Lévy flight finding in animal travel patterns. Note, however, that there is a significant difference between the data quality available in the animal and human travel patterns. Indeed, the mammalian data was available for short time periods for only a few animals, providing only a small number of observed individual displacements. Given the scarcity of data, precise statistical tools are needed to extract the proper fit. In contrast, the data analyzed in Ref. [11] as well as in this paper contain millions of displacements. Thus we are in a regime where typically traditional statistical tools, designed to deal with limited information, are less crucial. Yet, appropriate statistical tools can be used to explore the goodness
of the fit.
In this respect, it is often believed that statistical methods can validate a particular fit. The truth is, as emphasized in a recent publication [10], that these tools can only tell if a particular fit is consistent with the data, and rule some fits out, rather than validate a particular fit. A second important observation is that, given the high interest devoted to power laws, recently the issue of fitting a power law has been addressed in detail, developing the proper statistical tools to address the goodness of the fit [7-10]. The same tools are not available for truncated power laws, however, thus limiting the available methods to address their statistical relevance. In general we find that all the fits that we used in the paper pass the Kolmogorov-Smirnov test for the goodness of fit (Sect. V.A) and that a power law offers a much better approximation overall than an exponential function (Sect. V.B). Note that given the vast amount of data and the really good fit offered by the truncated power law, this last conclusion is hardly surprising.

## A. Kolmogorov-Smirnov goodness of fit test

We tested whether the empirical data could come from the fitted distributions by performing a stringent variant of the Kolmogorov-Smirnov ( $K S$ ) goodness of fit test [10]. The KS statistics is a simple way to compare whether two distributions are the same. In this case, we use it to test the hypothesis: Could the empirically observed distributions come from the distribution found as its best fit. For this we generated synthetic data starting from the fitted distribution and then use the KS test to see whether the empirical data we have behaves as well as the synthetic data generated from the fitted distribution.

We use two variants of the KS statistics to compare empirical data with the fitted distribution and synthetic data with the fitted distribution. The first method is the standard KS statistics and is given by:

$$
\begin{equation*}
K S=\max (|F-P|) \tag{S19}
\end{equation*}
$$

where $F$ is the cumulative distribution of the best fit and $P$ is the cumulative distribution of the empirical or synthetic data. The regular $K S$ statistic is not very sensitive on the edges of the cumulative distribution. Hence, we also used the weighted KS statistics defined as:


FIG. S5: $K S$ test for Fig 1C $D_{1}$


FIG. S6: $K S_{W}$ test for Fig 1C $D_{1}$

$$
\begin{equation*}
K S_{W}=\max \frac{|F-P|}{\sqrt{P(1-P)}} \tag{S20}
\end{equation*}
$$

To test whether the empirical data behaves as good as the synthetic data we calculated the $K S$ and $K S_{W}$ statistics between the empirical data and its best fit and compared these values with those obtained by calculating $K S$ and $K S_{W}$ for 1,000 synthetic data sets generated from the best fit. If the values obtained for $K S$ and $K S_{W}$ for the empirical data behave as good or better than those obtained for the synthetic data, then we can conclude that the empirical data is statistically consistent with its best fit. The results of the $K S$ test can be summarized using a $p-$ value


FIG. S7: $K S$ test for Fig 1D $D_{1}$


FIG. S8: $K S_{W}$ test for Fig 1D $D_{1}$
by integrating the distribution of $K S$ values generated with the synthetic data from the value representing the empirical distribution. When integrating such distributions from left to right we can interpret the $p$-value as the probability that the observed data was the result of its best fit. A $p$-value close to 1 will indicate that the empirical distribution matches its best fit as good as synthetic data generated from the fit itself [10], whereas a relative small $p$-value (typically taken $p<0.01$ ) would suggest that the empirical distribution can not be the result of its best fit.

Passing the $K S$ test does not rule out the possibility that the empirical data could be fitted as well or even better with some other function. In such a case, given the size of our samples, we
believe that such an exercise would be technical rather than practical and that different functional forms will closely resemble each other on the range were the fit was made once the fitting parameters have been fixed.

1. KS statistics for Fig 1C, $D_{1}$ : Figure $\mathbf{S 5}$ compares the $K S$ values obtained for the empirical data presented in Fig 1C, $D_{1}$ of the paper with those obtained for 1,000 distributions of synthetic data generated to comply with Eq. (1). Figure S 6 shows the same for $K S_{W}$. In both cases we find that the empirical data passes the $K S$ test, in fact behaving better than the synthetic data. Indeed $p(K S)=1$ and $p\left(K S_{W}\right)=1$.
2. KS statistics for Fig 1D, $D_{1}$ : Figure S 7 compares the $K S$ values obtained for the empirical data presented in Fig 1D, $D_{1}$ of the paper with those obtained for 2000 distributions of synthetic data generated to comply with Eq. (3). Figure S 8 shows the same for $K S_{W}$. In both cases we find that the empirical data passes the $K S$ test, behaving as some of the best examples of the synthetic data, obtaining $p(K S)=0.62$ and $p\left(K S_{W}\right)=0.82$.

## B. Maximum Likelihood Estimates: Comparing power-laws and exponentials

The Maximum Likelihood Method is a powerful way of estimating the fitting parameters best describing a empirical distribution. The method can also be used to compare the relative likelihood of two fits. In this section we are interested in testing whether a broad distribution, such as a power-law, is a better fit than an exponential for many of the distributions presented on the paper. It is not our intention to claim that the distributions presented here are in fact power-laws but to build suggestive evidence testing if the data presented in the paper is better fitted by a broad distribution, such as the power-law, rather than a narrow distribution such as an exponential. In fact, as we discuss in the manuscript and the previous section, we believe that the best fit to our data is given by a truncated power law rather than a simple power law. We do not present here the maximum likehood estimate of the truncated power-law fit because being its cumulative a Whittaker function, it is not readily to be handled with this method. For those interested in the accuracy of the best fit we suggest reading section VII A of this supplementary material.

The details of the maximum likelihood method have been widely published. Those interested in performing such fits could find help in [10] and [7].


FIG. S9:

Testing for broad and narrow distributions: The relative likelihood between two distributions can be calculated from its relative weights

$$
\begin{equation*}
W_{i}=\frac{e^{-\Delta_{i}}}{e^{-\Delta_{1}}+e^{-\Delta_{2}}}, \tag{S21}
\end{equation*}
$$

where $\Delta_{i}=\min \left(A I C_{i}-A I C_{\text {min }}\right)$. Here $A I C$ is the Akaike information criteria associated with the power-law $(i=1)$ and exponential $(i=2)$ fits. The AIC can be expressed as a function of the log-likelihood [7]

$$
A I C_{i}=2 \log \max \left(L_{i}\right)+2 K_{i}
$$

where $K_{i}$ is the number of parameters used in the fit and $L_{i}$ is the likehood of a particular value of a fitting parameter.

Here we test two fits, a power-law $A x^{-\beta}$ and an exponential $A e^{-\mu x}$, where $A$ is a normalization constant. In Fig. S9 we present the results of the Maximum Likelihood Fits for the distributions introduced in Fig. 1C and Fig. 1D of the paper. Note, we use the data log binned data, which has been recommended as the most accurate plotting method for this kind of analysis [8]. In each case we find $W_{\text {pow }} \gg W_{\text {exp }}$ indicating that a power law is a more likely fit than an exponential, indicating that the distribution of displacements, as well as $r_{g}$, are better approximated by a broad rather than a narrow distribution.
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only mean that morale at places such as the Environmental Protection Agency will improve.
So far, McCain and Obama have set out relatively different platforms on science and technology issues. For his part, Obama has yet to give a substantive speech on science issues, as Clinton did on the 50th anniversary of Sputnik. But he has adopted many of the traditional Democratic platforms, such as increasing federal funding for biomedical research and improving the jobs pipeline for young scientists. He has put a strong emphasis on the importance of technology for improving the lives of everyday Americans and their access to government. And he has been remarkably successful at using the Internet to turn his supporters into active participants in the political process. An Obama administration might well mean that more technologically savvy people will be drawn into public life - and perhaps that more young people will choose science and engineering careers.

In contrast, McCain has revealed few details of his science and
> "There can be little doubt that the next US administration will be more science-friendly than the current one."
technology agenda. But in some areas he has been quite outspoken. Last week, for example, he called for a new treaty to reduce the US and Russian nuclear arsenals, suggesting he would lower the number of weapons even beyond the cuts planned by the Bush administration. And on 12 May he outlined a detailed plan for controlling greenhouse-gas emissions - the centrepiece of which is a cap-and-trade system of the type he has been advocating since at least 2003, when he co-sponsored the first meaningful bill on that subject.
In the end, the main factor is not how Obama or McCain feels about specific science-related issues. What American voters deserve to know is how each candidate's mind works. Does he listen to a handful of ideology-driven advisers in making key decisions? Or does he look facts in the face and base his conclusions on all the available evidence? If it's the latter, then that is the candidate who is in sync with science at a level far more meaningful than any immediate argument over research budgets or competitiveness.

## A flood of hard data

## Social scientists have a new handle on group behaviour - but its causes remain a challenge.

Every human being makes choices, takes action and is affected - by the environment in a way that seems utterly idiosyncratic. Yet in the aggregate, as the British philosopher John Stuart Mill put it a century and a half ago, human events "most capricious and uncertain" can take on "a degree of regularity approaching to mathematical". A case in point is the analysis of mobile-phone data discussed by González et al. on page 779 of this issue. It reveals just such a mathematical regularity in the seemingly unpredictable way people move around during their daily lives (see also page 714).

As remarkable as this result is - and it is still not completely understood - the research is just as notable for its methodology. Social scientists have long struggled with a paucity of hard data about human activities; people's self-reporting about their social interactions, say, or their movement patterns is labour-intensive to collect and notoriously unreliable. In this case, the researchers obtained objective data on individuals' movements from mobile-phone networks (albeit without access to any individual's identity, for privacy reasons). This gave them a data set of proportions almost unheard of for such a complex aspect of behaviour: more than 16 million 'hops' for 100,000 people. The resulting statistics show a strikingly small scatter, giving grounds for confidence in the mathematical laws they disclose.

The mobile-phone technique is simply the latest example of how modern information technologies are giving social scientists the power to make measurements that are often as precise as those in the 'hard' sciences. By analysing e-mail transmissions, for example, or doing automated searches of publication databases,

## "The goal of social

science is not simply to understand how people behave in large groups, but to understand what motivates individuals to behave the way they do."
social scientists can collect detailed information on the network structure of scientific collaborations and other social interactions. And by allowing their subjects to interact online, researchers can do large-scale studies of, for example, the role of social interactions in opinion formation, complete with control groups and tuneable parameters.

It's not an overstatement to say that these tools are fostering a whole new type of social science - with applications that go well beyond the conventional boundaries of the field. There is sure to be commercial interest in the detailed patterns of usage for portable electronics, for example, and the nature of mass human movement could inform urban planning and the development of transportation networks. Epidemiologists, meanwhile, will no longer be forced to work with highly oversimplified models of infection rates and disease spread: recent work has clarified how the transmission of disease depends on the precise structural details of the network of person-to-person contacts.

For all their promise, making sense of these new data sets requires a rather different set of statistical skills than those needed in conventional social science - which may be one reason why studies such as that by González et al. are so often conducted by researchers trained in the physical sciences. To some extent this 'physicalization' of the social sciences is healthy for the field; it has already brought in many new ideas and perspectives. But it also needs to be regarded with some caution.

As many social scientists have pointed out, the goal of their discipline is not simply to understand how people behave in large groups, but to understand what motivates individuals to behave the way they do. The field cannot lose focus on that - even as it moves to exploit the power of these new technological tools, and the mathematical regularities they reveal. Comprehending capricious and uncertain human events at every level remains one of the most challenging questions in science.


Animal behaviour is an endless challenge to mathematical modellers. In the first of two features, Mark Buchanan looks at how a mathematical principle from physics might be able to explain patterns of movement. In the second, Arran Frood asks what current models can teach us about ecological networks half a billion years old.

# THE MATHEMATICAL MIRROR TO ANIMAL NATURE 

Food is not, in general, spread equally around the world; it comes in lumps. Foragers thus need a strategy for finding those lumps. One appealing option is a Lévy flight - a mathematical concept used in physics. Lévy flights are many-legged journeys in which most of the legs are short, but a few are much longer. They are found in some sorts of diffusion, in fluid turbulence, even in astrophysics. In animal behaviour, the longer the flight, the farther afield a creature will get, offering a way to efficiently exploit food nearby but also to discover sources farther away.
"The pattern captures what biologists often
notice," says behavioural ecologist David Sims of the Marine Biological Association Laboratory in Plymouth, UK. "Animals often take lots of short steps in a localized area before making long jumps to new areas."

But just because it makes qualitative sense doesn't mean it is a mathematical key to the real world. Hard evidence is needed to show that the pattern is a real Lévy flight, in which the frequency of steps of given distances is firmly constrained. And this evidence is what physicist Gandhimohan Viswanathan, then a graduate student at Boston University in Massachusetts, and his colleagues seemed to find in 1996.

Albatrosses soar over tremendous distances as they circle the oceans, alighting here or there to feed on squid, fish or krill before heading off again. Observers had thought the foraging was random; but any hidden pattern would be evident only on the scale of seas and oceans. It was this large pattern that Viswanathan, now at the Federal University of Alagoas in Brazil, decided to look for, using electronic logging data gathered by field ecologists at the British Antarctic Survey (BAS) in Cambridge.

Viswanathan and his colleagues found a scale-free fractal-like pattern in the data ${ }^{1}$, just what a Lévy flight ought to produce. Three years
later, they seemed to be on the track of a new principle of ecology when they showed that this way of moving is, under some conditions, theoretically the best way for animals to find scarce prey ${ }^{2}$. They and other researchers soon reported the same pattern in the movements of everything from reindeer and bumblebees to soil amoebas and the habits of fishermen ${ }^{3}$. The phenomenon is attracting more and more interest, and it seems to apply to more than just foraging. Research in this week's Nature shows that it applies to the movements of mobilephone users ${ }^{4}$ too (see page 779).

## Flights of fancy?

There's just one problem. Although other examples stand up to scrutiny, the one that started the field off does not, at least for now. There's a lesson in that. When modellers use data from the field, they have to be sure that the data really represent what they think they represent, and that they fit tightly to their model. The devil is in the detail, when sparse data can put almost all conclusions on shaky ground.

The case for Lévy flights by albatrosses ran into problems in 2004, when physicist Sergey Buldyrev, also of Boston University and one of Viswanathan's co-authors on the original albatross paper, analysed new data on albatross movements. The Lévy pattern didn't turn up. Revisiting the original data collected by the BAS
researchers, Buldyrev discovered that the longest flights recorded, which were crucial to the distinctive fractal fingerprint, might have been artefacts of the recording technique.
The original albatross data came from devices called immersion loggers attached to the birds' legs. The devices recorded the proportion of time in each quarter-minute that the birds sat on the sea surface. From these data, the researchers could then infer flights as periods during which the birds remained dry. From five birds, the researchers had obtained a total of 363 flight times, which seemed to show the Lévy pattern.

But Buldyrev wondered whether the longest periods of dry-leggedness - which always seemed to be the first and last in a bout of movement - might in fact record the birds sitting on their nests. The data had not been saying what the team thought they were saying. Finding that the Lévy pattern vanished when these data points were omitted, Buldyrev and his colleagues wrote up a manuscript and sent a draft to ecologist Richard Phillips at the BAS. Phillips, working with ecological modeller Andrew Edwards, also at the BAS, confirmed that there was no support for Lévy flights. Later, when they discovered that some of the albatrosses also had location trackers fitted to them, the BAS team proved that the birds
weren't moving during the alleged long flights. "I was disappointed," says Viswanathan, "but also curious, surprised and perplexed."

The Lévy flight notion took another blow last October, when the Boston and Cambridge groups collaborated to publish a comprehensive reanalysis of the original albatross data, including an analysis of a new data set and a reconsideration of earlier studies of deer and bumblebees ${ }^{5}$. They found that the deer and bumblebee data were also ambiguous - the deer data, for example, actually reflected time spent cropping and processing food at a particular feeding site, rather than time spent moving between sites. Using improved statistical techniques, the teams found that none of the data offered strong support for the Lévy flight pattern. The results, they say, "question the strength of the empirical evidence for biological Lévy flights."

It looked like a simple tale of problematic data corrected. But later last year, Sims and his colleagues presented strong evidence for Lévy-like patterns in the foraging of numerous marine predators, including sharks, turtles and penguins ${ }^{6}$. They used what all researchers agree are more sophisticated statistical methods, and much larger data sets. Sims and others now suggest that the data really do point to Lévy flights for a variety of animals, including humans.

Not everyone yet agrees with this position. But they do agree that the episode illustrates the difficulties inherent in identifying statistical patterns with limited data. The difference between a Lévy flight and a more familiar form of random walk, brownian motion, is the distribution of steps of different lengths. In brownian
motion, as seen in the jittering of a pollen grain buffeted on all sides by invisible molecules, the distribution of distances follows a bell-shaped curve, so the size of the next step is at least crudely predictable - it is never 10 or 100 times bigger than the average, for example.

## Doing the Lévy walk

A Lévy flight is a similar sort of random walk - but the distribution of distances is different. For example, the probability of large steps of size D might fall off in proportion to $\mathrm{d}^{\gamma}$, with $\gamma$ being a number somewhere between
> "It was wrong, yet it turned out to be fruitful because it led to other ideas that we now think are correct." - Eugene Stanley 1 and 3. This distribution, in what is known as a power law, gives more frequent long steps than a bell curve, and produces a pattern characterized by lots of smaller movements broken episodically by long excursions.
Diagnosing a true Lévy flight means showing that the powerlaw distribution holds. There is a simple statistical approach to this. First 'bin the data': that is, count up the events that fall within each small range of distances to get a measure of the way the probability of differing distances is distributed. If a power law holds, the relationship between the logarithm of this probability distribution and the logarithm of the distance will be linear. Hence, if the log of the first is charted against the log of the second, you'll get a straight line.

As Edwards points out, however, this technique can lead to trouble. "It's well known that $\log -\log$ axes tend to make relationships look straight." The problem is at its worst when data are in short supply. A more rigorous approach, he says, is to decide mathematically which of two possible distributions, say a power law or an exponential, the data fit better. But such


Dry of foot, but not flying.


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